Eur. Phys. J. C 9, 213–219 (1999)
DOI 10.1007/s100529900039

THE EUROPEAN
PHYSICAL JOURNAL C

Inclusive decays and lifetimes of doubly-charmed baryons

B. Guberina[a], B. Melić[b], H. Štefančić[c]

Theoretical Physics Division, Rudjer Bošković Institute, P.O. Box 1016, HR-10001 Zagreb, Croatia

Received: 21 January 1999 / Published online: 20 May 1999

Abstract. The analysis of singly-charmed hadrons has been extended to the case of doubly-charmed baryons, Ξ_{cc}^{++}, Ξ_{cc}^{+} and Ω_{cc}^{+}. Doubly-charmed baryons are described as a system containing a heavy cc diquark and a light quark, as in the case of a heavy–light meson. This leads to preasymptotic effects in semileptonic and nonleptonic decays that are essentially proportional to the meson wave function. Interplay between preasymptotic effects in semileptonic and/or nonleptonic decay rates leads to very clear predictions for semileptonic branching ratios and lifetimes of doubly-charmed baryons.

1 Introduction

Weak decays of heavy hadrons [1–4] present a very rich field of phenomena owing to the complexity of confinement. Being the bound states of heavy-quark and light constituents (mesons and singly-charmed or bottom baryons) or even of two heavy quarks and one light constituent (doubly-charmed or bottom baryons), heavy hadrons contain soft degrees of freedom which generate nonperturbative power corrections, such as the destructive and/or constructive Pauli interference and the W exchange/annihilation between a light constituent and a quark coming from the heavy-quark decay.

The inclusive decay rates and lifetimes of charmed mesons that have been calculated in the last decade are fairly reliable. The overall picture emerging is qualitatively satisfactory, and the lifetime hierarchy predicted for singly-charmed baryons has been found to be in agreement with present experiments [4]. The difference in lifetimes (a factor of 2–3) between D^+ and D^0 mesons, which is due to the negative Pauli interference preasymptotic effect, has also been explained, a long time ago [5–7].

The numerical calculations performed in the mid-eighties [8,9] provided us with predictions of a lifetime pattern that has recently been confirmed by experiment. This success is rather surprising, since with the advent of the systematic operator product expansion (OPE) [1] and heavy quark effective theory (HQET) [10], it has become clear that the charmed-quark mass is not heavy enough for the m_c^{-1} expansion to be trustworthy. Nevertheless, it seems that if one systematically employs field-theory methods to the very end of the calculation (up to the hadronic wave function, for which we have to rely upon some phenomenological models), one is able to make clear predictions that can be compared with present and future experiments, and can possibly disentangle various preasymptotic effects.

On the other hand, the inverse bottom-quark mass appears to be a good expansion parameter in bottom decays. However, the role of four-quark operators is negligible there (effects of the $O(1\%)$), leaving charmed-hadron decays as a playground for studying such effects and for testing the possible violation of the quark–hadron duality.

In this paper we extend the analysis of singly-charmed-baryon decays and lifetimes [11] to the case of doubly-charmed baryons. Recently [12], a rather phenomenological approach using effective constituent–quark masses and a fit of singly-charmed-baryon decays has been employed to study doubly-charmed-baryon decays. We, however, have used a systematic field-theory approach to the very end in order to be consistent with the previous treatment of singly-charmed hadrons. We have also included the preasymptotic effects in semileptonic decay rates of doubly-charmed baryons and calculated all decay rates at the Cabibbo subleading level. We show that preasymptotic effects dramatically change the simple spectator picture, and lead to a very clear pattern of semileptonic branching ratios and lifetimes.

2 Preasymptotic effects and the wave function in doubly-charmed-baryon decays

Using the optical theorem, the inclusive decay width of a hadron H_{cc} with mass $M_{H_{cc}}$ containing two heavy c quarks can be written as

$$\Gamma(H_{cc} \to f) = \frac{1}{2M_{H_{cc}}} 2\,\mathrm{Im}\langle H_{cc}|\hat{T}|H_{cc}\rangle\,, \tag{1}$$

where $\hat{T}$ is the transition operator

[a] guberina@thphys.irb.hr
[b] melic@thphys.irb.hr
[c] shrvoje@thphys.irb.hr

$$\hat{T} = \mathrm{i} \int \mathrm{d}^4 x T\{L_{\mathrm{eff}}(x), L^{\dagger}_{\mathrm{eff}}(0)\}. \quad (2)$$

In the following we use the OPE, which is based on the assumption that the energy release in the decay of a c quark is large enough. This implies that momenta flowing through internal lines are also large, and therefore justify the OPE.

The general formula for the decay is given by [1–3]

$$\begin{aligned} \Gamma(H_{cc} \to f) = \frac{G_{\mathrm{F}}^2 m_c^5}{192\pi^3} |V|^2 \frac{1}{2M_{H_{cc}}} \Bigg\{ & c_3^f \langle H_{cc}|\bar{c}c|H_{cc}\rangle \quad (3) \\ & + c_5^f \frac{\langle H_{cc}|\bar{c} g_s \sigma^{\mu\nu} G_{\mu\nu} c|H_{cc}\rangle}{m_c^2} \\ & + \sum_i c_6^f \frac{\langle H_{cc}|(\bar{c}\Gamma_i q)(\bar{q}\Gamma_i c)|H_{cc}\rangle}{m_c^3} + O(1/m_c^4) \Bigg\}. \end{aligned}$$

Here c_3^f and c_5^f are Wilson coefficient functions which are known at tree level and one-loop order, respectively [1–3]. V represents appropriate matrix elements of the Cabibbo–Kobayashi–Maskawa (CKM) matrix.

Let us calculate the semileptonic decay rates first. The main contribution is expected to come from a quark-decay-type diagram; the decay is proportional to $\langle H_{cc}|\bar{c}c|H_{cc}\rangle$, and is given, to $O(m_c^2)$, as

$$\begin{aligned} \Gamma_{\mathrm{SL}}^{\mathrm{dec}}(H_{cc}) = & 2\frac{G_F^2}{192\pi^3} m_c^5 (\mathrm{c}^2 \eta_{SL}(x) P_0(x) + \mathrm{s}^2 \eta_{\mathrm{SL}}(0)) \\ & \times \left(1 - \frac{1}{2}\frac{\mu_\pi^2(H_{cc})}{m_c^2} + \frac{1}{2}\frac{\mu_G^2(H_{cc})}{m_c^2}\right). \quad (4) \end{aligned}$$

Throughout the paper we use the abbreviations s^2 and c^2 for $\sin^2\theta_c$ and $\cos^2\theta_c$ (θ_c is the Cabibbo angle). Here $\mu_\pi^2(H_{cc})$ and $\mu_G^2(H_{cc})$ parametrize the matrix elements of the kinetic energy and the chromomagnetic operators, respectively. Their determination will be discussed later.

The next contribution comes from the dimension-five operator

$$\Gamma_{\mathrm{SL}}^G(H_{cc}) = 2\frac{G_F^2}{192\pi^3} m_c^5 \left(\mathrm{c}^2 P_1(x) + \mathrm{s}^2\right) \left(-2\frac{\mu_G^2(H_{cc})}{m_c^2}\right). \quad (5)$$

Note that in both (4) and (5) there is an additional factor 2 coming from the decays of *two* c quarks in the doubly-charmed baryon.

The phase-space corrections P_0 and P_1 are cited explicitly in the Appendix. The radiative QCD correction η_{SL} [13,14] is given by

$$\eta_{\mathrm{SL}}(x) = 1 - \frac{2}{3}\frac{\alpha_S}{\pi} g(x), \quad (6)$$

where for $g(x)$ we have

$$g(x) = \pi^2 - \frac{25}{4} + x(18 + 8\pi^2 + 24\ln x), \quad (7)$$

and $x = m_s^2/m_c^2$. Leptons are taken to be massless.

Recently, Voloshin noticed [15] that preasymptotic effects in *semileptonic* inclusive decays could be very large because of the constructive Pauli interference; the result up to the CKM matrix element is given by

$$\tilde{\Gamma}_{\mathrm{SL}} = \frac{G_F^2}{12\pi} m_c^2 (4\sqrt{\kappa} - 1) 5|\psi(0)|^2. \quad (8)$$

Here κ is a correction due to the hybrid renormalization of the effective Lagrangian, and it takes care of the evolution of L_{eff} from m_c down to the typical hadronic scale $\mu \sim$ 0.5–1 GeV. The factor 5 in front of $|\psi(0)|^2$ reflects the spin structure of doubly-charmed baryons. The baryon wave function $\psi(0)$ will be discussed later.

The total semileptonic rate for one lepton species is given by

$$\Gamma_{\mathrm{SL}}(H_{cc}) = \Gamma_{\mathrm{SL}}^{\mathrm{dec}}(H_{cc}) + \Gamma_{\mathrm{SL}}^G(H_{cc}) + \Gamma_{\mathrm{SL}}^{\mathrm{Voloshin}}(H_{cc}), \quad (9)$$

where

$$\begin{aligned} \Gamma_{\mathrm{SL}}^{\mathrm{Voloshin}}(\Xi_{cc}^{++}) &= 0, \\ \Gamma_{\mathrm{SL}}^{\mathrm{Voloshin}}(\Xi_{cc}^{+}) &= \mathrm{s}^2 \tilde{\Gamma}_{\mathrm{SL}}, \\ \Gamma_{\mathrm{SL}}^{\mathrm{Voloshin}}(\Omega_{cc}^{+}) &= \mathrm{c}^2 \tilde{\Gamma}_{\mathrm{SL}}. \quad (10) \end{aligned}$$

In view of the significant preasymptotic effects in the SL decay rates of singly-charmed baryons, one can expect a large Pauli-interference contribution in the semileptonic decay rate of the Ω_{cc}^+ baryon (ccs quark structure), where that contribution is present at the leading Cabibbo level.

Nonleptonic decay rates are slightly more complicated, since in the final state, the lepton pair is substituted by a quark pair. The contributions analogous to (4) and (5) are (including $O(m_c^3)$ corrections)

$$\begin{aligned} \Gamma_{\mathrm{NL}}^{\mathrm{dec}}(H_{cc}) = 2\frac{G_F^2}{192\pi^3} m_c^5 (c_-^2 + 2c_+^2)[((\mathrm{c}^4 + \mathrm{s}^4) P_0(x) + \\ \mathrm{c}^2\mathrm{s}^2)\eta_{\mathrm{NL}}(x) + \mathrm{c}^2\mathrm{s}^2 \tilde{P}_0(x)\tilde{\eta}_{\mathrm{NL}}(x)] \\ \times \left[1 - \frac{1}{2}\frac{\mu_\pi^2(H_{cc})}{m_c^2} + \frac{1}{2}\frac{\mu_G^2(H_{cc})}{m_c^2}\right], \quad (11) \end{aligned}$$

$$\begin{aligned} \Gamma_{\mathrm{NL}}^G(H_{cc}) = 2\frac{G_F^2}{192\pi^3} m_c^5 \{(2c_+^2 + c_-^2)[((\mathrm{c}^4 + \mathrm{s}^4) P_1(x) + \\ \mathrm{c}^2\mathrm{s}^2)\eta_{\mathrm{NL}}(x) + \mathrm{c}^2\mathrm{s}^2 \tilde{P}_1(x)\tilde{\eta}_{\mathrm{NL}}(x)] \\ + 2(c_+^2 - c_-^2)[((\mathrm{c}^4 + \mathrm{s}^4) P_2(x) + \mathrm{c}^2\mathrm{s}^2)\eta_{\mathrm{NL}}(x) + \\ \mathrm{c}^2\mathrm{s}^2 \tilde{P}_2(x)\tilde{\eta}_{\mathrm{NL}}(x))]\} \left(-2\frac{\mu_G^2(H_{cc})}{m_c^2}\right). \quad (12) \end{aligned}$$

Radiative corrections to the nonleptonic decay, $\eta_{\mathrm{NL}}(x)$ and $\tilde{\eta}_{\mathrm{NL}}(x)$, are far more complicated than analogous corrections (6) and (7) to the semileptonic decay; the reader is referred to the original paper where they were first calculated [16].

Again, the preasymptotic effects are expected to contribute significantly to the total nonleptonic decay rate. They are given by

$$\Gamma^{\mathrm{ex}} = \frac{G_F^2}{2\pi} m_c^2 \left[c_-^2 + \frac{2}{3}(1 - \sqrt{\kappa})(c_+^2 - c_-^2)\right] 5|\psi(0)|^2,$$

$$\begin{aligned}\Gamma_-^{\rm int} &= \frac{G_F^2}{2\pi} m_c^2 \left[-\frac{1}{2} c_+ (2c_- - c_+) \right. \\ &\quad \left. -\frac{1}{6}(1-\sqrt{\kappa})(5c_+^2 + c_-^2 - 6c_+c_-) \right] 5|\psi(0)|^2 \,, \\ \Gamma_+^{\rm int} &= \frac{G_F^2}{2\pi} m_c^2 \left[\frac{1}{2} c_+ (2c_- + c_+) \right. \\ &\quad \left. -\frac{1}{6}(1-\sqrt{\kappa})(5c_+^2 + c_-^2 + 6c_+c_-) \right] 5|\psi(0)|^2 \,. \quad (13)\end{aligned}$$

An explicit calculation leads to the following nonleptonic decay rates:

$$\begin{aligned}\Gamma_{\rm NL}(\Xi_{cc}^{++}) &= \Gamma_{\rm NL}^{\rm dec}(\Xi_{cc}^{++}) + \Gamma_{\rm NL}^{G}(\Xi_{cc}^{++}) \\ &\quad + \{(\mathrm{c}^4+\mathrm{s}^4)P_{\rm int}(x) + \mathrm{c}^2\mathrm{s}^2(1+\tilde{P}_{\rm int}(x))\}\Gamma_-^{\rm int} \,, \\ \Gamma_{\rm NL}(\Xi_{cc}^{+}) &= \Gamma_{\rm NL}^{\rm dec}(\Xi_{cc}^{+}) + \Gamma_{\rm NL}^{G}(\Xi_{cc}^{+}) \\ &\quad + (\mathrm{c}^4 P_{ex}(x) + \mathrm{c}^2\mathrm{s}^2)\Gamma^{\rm ex} \\ &\quad + (\mathrm{s}^4 P_{\rm int}(x) + \mathrm{c}^2\mathrm{s}^2)\Gamma_+^{\rm int} \,, \\ \Gamma_{\rm NL}(\Omega_{cc}^{+}) &= \Gamma_{\rm NL}^{\rm dec}(\Omega_{cc}^{+}) + \Gamma_{\rm NL}^{G}(\Omega_{cc}^{+}) \\ &\quad + (\mathrm{c}^4 + \mathrm{c}^2\mathrm{s}^2 P_{\rm int}(x))\Gamma_+^{\rm int} \\ &\quad + (\mathrm{c}^2\mathrm{s}^2 P_{\rm ex}(x) + \mathrm{s}^4)\Gamma^{\rm ex} \,. \quad (14)\end{aligned}$$

All corrections P and $\tilde{P}$ are given explicitly in the Appendix.

An important remark to be made here concerns the mass parameters in the calculation of the matrix elements μ_π^2 and μ_G^2. Whenever we perform an expansion, which is essentially a field-theoretic procedure (either the OPE for the transition operator $\hat{T}$, or the HQET expansion in the case of the $\bar{c}c$ operator), the expansion parameter is always the current heavy-quark running mass m_c. On the other hand, in the calculation of the matrix elements, which is performed within quark models, it is more appropriate to use constituent quark masses m^*.

Following this procedure, we give the expressions for μ_π^2 and μ_G^2. For μ_π^2, we have

$$\mu_\pi^2 = m_c^2 v_c^2 = \left(\frac{m_q^* T}{2m_c^{*2} + m_c^* m_q^*} + \frac{T}{2m_c^*} \right) m_c^2 \,, \qquad (15)$$

where v_c is the average heavy-quark velocity in the ccq baryon, m_c^* and m_q^* are constituent masses of the heavy and the light quark, respectively, and T is the average kinetic energy of the light quark and the heavy diquark. The precise description of this calculation is given in [12] and relies upon some phenomenological features of the meson potential.

The contributions to the μ_G^2 operator, which are connected to the matrix element of the chromomagnetic operator, can be divided into two parts. The first part includes effects coming from the heavy–light chromomagnetic interaction and these contributions can also be found in the singly-charmed baryon Ω_c^+. The second part comprises effects originating within the heavy diquark, i.e., heavy–heavy chromomagnetic interactions. These effects are new [12,17] and characteristic of doubly-charmed baryons. Their estimation relies upon the nonrelativistic QCD model calculation [12,17–19]. The final expression is

$$\mu_G^2 = \frac{2}{3}(M_{ccq}^* - M_{ccq})m_c - \left(\frac{2}{9} g_S^2 \frac{|\phi(0)|^2}{m_c^*} + \frac{1}{3} g_S^2 \frac{|\phi(0)|^2}{m_c} \right) \,, \qquad (16)$$

where the first term describes the heavy diquark–light quark hyperfine interaction, while the second and the third terms correspond to the interaction of two heavy c quarks in a diquark state. They are of the "chromomagnetic" and "Darwin" types, respectively. In (16), M_{ccq} is the mass of the doubly-charmed baryon, M_{ccq}^* is the mass of its 3/2-spin counterpart, and $\phi(0)$ is the wave function of the cc pair in the heavy diquark, i.e., $|\phi(0)|^2$ is the probability for these two heavy quarks to meet at one point.

In the calculations above, we have used only field theory up to the hadronic matrix elements. The results are expressed in terms of the baryon wave function $\psi(0)$ and the matrix elements of the kinetic and chromomagnetic operators, which are μ_π^2 and μ_G^2, respectively. The use of the usual singly-charmed-baryon wave function $\Psi(0)$, as given in [11], would be premature, since intuitively, one expects a two-heavy-quark system to behave differently from the single-heavy-quark one.

In the case of singly-charmed baryons, the heavy quark is stationary in the center of the baryon, and the other two light quarks are moving around. Their spin and color charges are correlated (in order to have the appropriate spin and color structure of the entire baryon), but their spatial motion is not. In this way, one has a three-body picture of the baryon containing a single heavy quark, and one should use the baryonic wave function $\Psi(0)$ accordingly.

In the case of doubly-charmed baryons, one assumes that two heavy quarks are strongly bound into a color antitriplet state. As far as the light quark is concerned, the bound state of two cc quarks appears as a pointlike diquark object [20–22]. Thus, in the heavy-quark limit, which can (presumably) still be applied in our case ($m_c > \Lambda_{\rm QCD}$), a doubly-charmed baryon appears to consist of a heavy diquark and a light quark, forming a meson-like state. Therefore, one expects that the doubly-charmed-baryon wave function (which has to be considered as the light-quark wave function at the origin of the cc diquark) behaves essentially as the mesonic wave function.

We use the derivation of hyperfine splittings of mesons calculated in the constituent nonrelativistic quark model by De Rújula, et al. [23,24] to obtain the following relation between the wave functions of the doubly-charmed baryon and the D meson:

$$|\psi(0)|^2 = \frac{2}{3}|\psi(0)|_D^2 = \frac{2}{3}\frac{f_D^2 M_D \kappa^{-4/9}}{12} \,. \qquad (17)$$

The factor 2/3 comes from the different spin content of doubly-charmed baryons; i.e., the cc diquark forms the spin-1 color antitriplet state. The baryonic wave function squared in (17) is directly proportional to the D meson decay constant, f_D, squared. The factor $\kappa^{-4/9}$ is the effect of the hybrid renormalization, which accounts for the fact

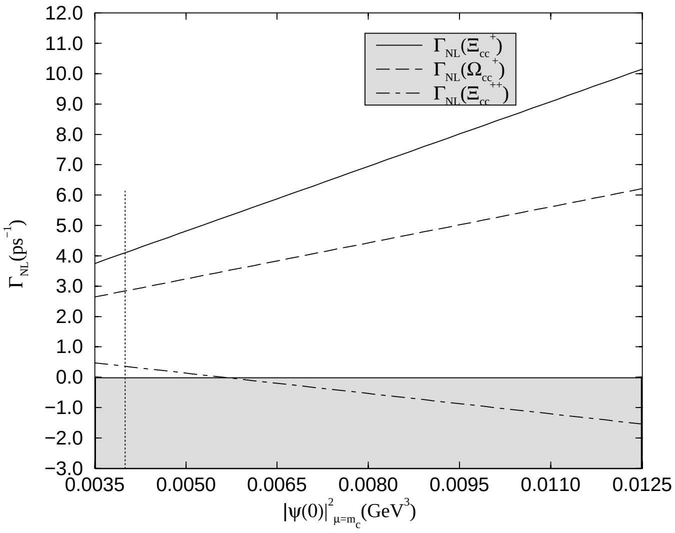

Fig. 1. Dependence of nonleptonic decay widths on the squared wave function. The shaded area represents the unphysical region of negative $\Gamma_{\rm NL}$, stressing the problem of choice of the wave function. The vertical line represents the squared value of the wave function (corresponding to $f_D = 170\,$MeV) used in the calculations.

Table 1. Predictions for nonleptonic widths, semileptonic widths, semileptonic branching ratios (for one lepton species) and lifetimes of doubly-charmed baryons for the values of parameters $m_c = 1.35$ GeV, $\mu = 1$ GeV, $\Lambda_{\rm QCD} = 300$ MeV, $f_D = 170$ MeV.

	Ξ_{cc}^{++}	Ξ_{cc}^{+}	Ω_{cc}^{+}
Nonleptonic widths in ps^{-1}			
$\Gamma_{\rm NL}$	0.345	4.158	2.859
Semileptonic widths in ps^{-1}			
$\Gamma_{\rm SL}$	0.151	0.173	0.603
Semileptonic branching ratios in%			
$BR_{\rm SL}$	23.4	3.9	14.9
Lifetimes in ps			
τ	1.55	0.22	0.25

that f_D is measured at the scale proportional to m_c ($\kappa = 1$), and that one has to evolve f_D down to the hadronic scale $\mu = 0.5 - 1$ GeV.

The choice of the mesonic wave function proportional to f_D, instead of the singly-charmed-baryon wave function, $|\Psi(0)|^2 \sim F_D^2$, where F_D is the static value of the D meson decay constant, also seems to be consistent numerically. In Fig. 1 we have displayed the dependence of $\Gamma_{\rm NL}(H_{cc})$ on $|\psi(0)|^2$ in the large range of the f_D values. In our numerical calculation, we use $f_D = 170$ MeV as a central value. This value is consistent with both QCD lattice calculations [25] and QCD sum rule calculations [26, 27]. In the case of $\Gamma_{\rm NL}(\Xi_{cc}^{++})$ there is a negative Pauli interference which cancels the contribution coming from the decay-type diagram (11) and the chromomagnetic operator (12). This case is very similar to that of the negative Pauli interference in the D^+ decay, where it competes with the decay-diagram contribution. For f_D large enough, the nonleptonic and total rates in both D^+ and Ξ_{cc}^{++} decays become negative (see Fig. 1). However, a reasonable choice of f_D gives positive results.

In view of these facts and results, we may conclude that the phenomenological rule of using f_D in mesonic systems and its static value F_D in baryonic systems, employed first in singly-charmed hadrons [1], can be successfully extended to the consideration of doubly-charmed baryons. Not doing so, and taking F_D instead of f_D, would lead us to the unphysical region.

Thus, our result is a confirmation of the above-mentioned phenomenological rule at the same, qualitative level. Taking this rule as postulated for singly-charmed hadrons, we can interpret our results as an extension of the same rule into the doubly-charmed sector. Also, we can generalize the rule to some extent. We see that the use of f_D is required in singly-charmed mesons and doubly-charmed baryons, while F_D is used in singly-charmed baryons [11]. So, it is allowed to say that f_D should be used in systems with two-body dynamics (in the ccq baryon case, a heavy diquark and a light quark), and F_D in systems with three-body dynamics. It is important to stress that these considerations and conclusions are of purely phenomenological origin, i.e., they have no direct justification in field theory.

3 Semileptonic inclusive rates and lifetimes – results and discussions

In numerical calculations, we use the following set of parameters, which closely follows the set used in [11]: For $\Lambda_{\rm QCD} = 300$ MeV, the Wilson coefficients are $c_+ = 0.73$ and $c_- = 1.88$. The charmed-quark mass is taken to be $m_c = 1.35$ GeV, and for the strange-quark mass we use $m_s = 150$ MeV. The value of the average kinetic energy T, appearing explicitly in (15), is taken from [12] to be $T = 0.4$ GeV, and the light- and heavy-quark constituent masses are $m_q^* = 0.3$ GeV and $m_c^* = 1.6$ GeV, respectively. The numerical value of the diquark wave function is also taken from [12]: $|\phi(0)| = 0.17\,{\rm GeV}^{3/2}$. The numerical values for masses of doubly-charmed baryons are taken from [28].

As far as the $\Lambda_{\rm QCD}$ dependence is concerned, in the range $\Lambda_{\rm QCD} \sim 200 - 300$ MeV, the lifetimes of Ξ_{cc}^{+} and Ω_{cc}^{+} are practically constant, and the lifetime of Ξ_{cc}^{++} is more sensitive to the value of $\Lambda_{\rm QCD}$; it is somewhat (10%) larger for $\Lambda_{\rm QCD} = 200$ MeV. The same is true for the μ dependence in the reasonable range $\mu \sim 0.5 - 1$ GeV. The lifetimes of Ξ_{cc}^{+} and Ω_{cc}^{+} stay almost constant with variation of μ, and the lifetime of Ξ_{cc}^{++} grows slowly with μ (by 18%) (see Fig. 2).

From Table 1, one can see that the Voloshin type of preasymptotic corrections in the semileptonic decay rates of Ω_{cc}^{+} is significant, contributing at the Cabibbo leading

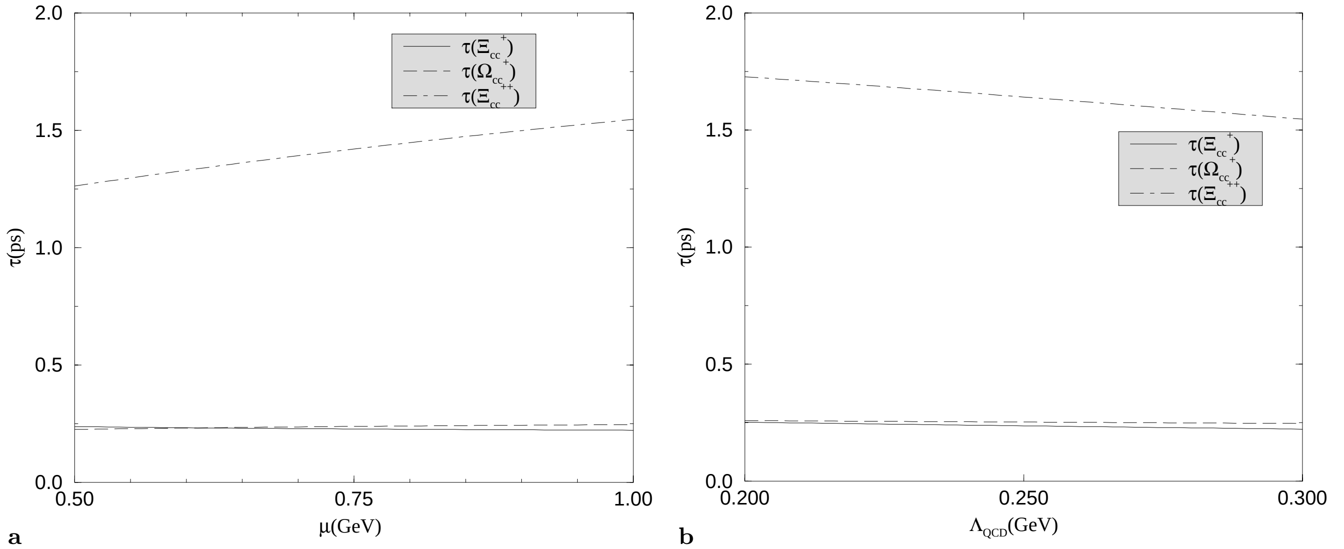

Fig. 2a,b Dependence of lifetimes on the parameters μ and $\Lambda_{\rm QCD}$. Both pictures show the evident insensitivity of $\tau(\Omega_{cc}^{+})$ and $\tau(\Xi_{cc}^{+})$ under variation of μ and $\Lambda_{\rm QCD}$, and a small but notable sensitivity of $\tau(\Xi_{cc}^{++})$.

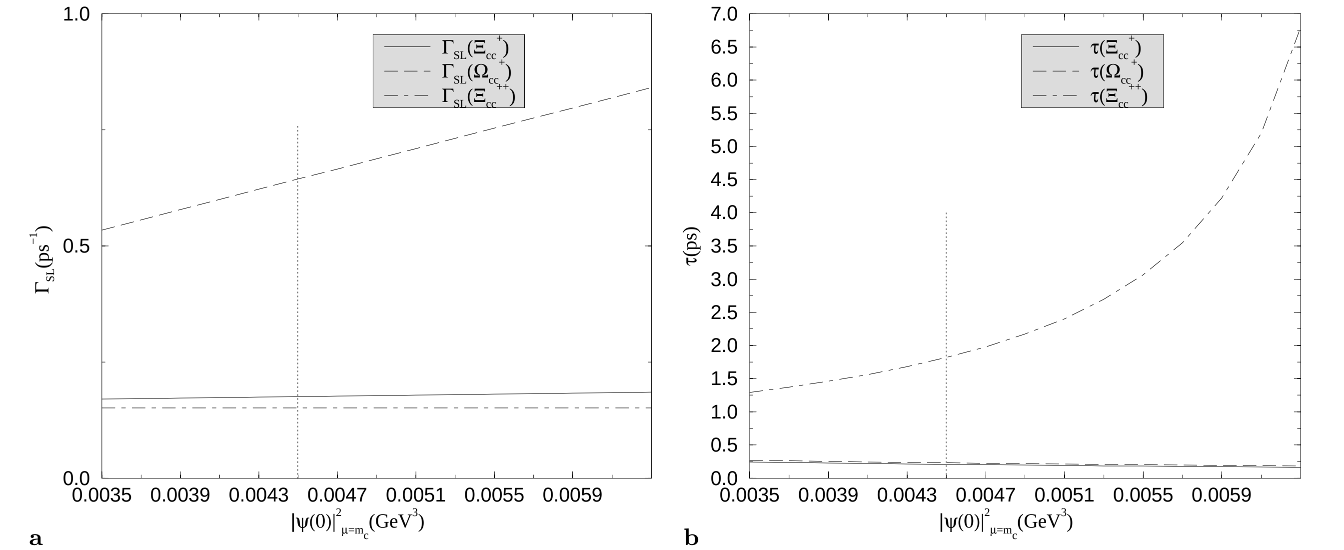

Fig. 3a,b Dependence of semileptonic decay widths and lifetimes on the squared value of the wave function. The vertical line represents the $|\psi(0)|^2$ used in calculations, and corresponds to $f_D = 170$ MeV. The second picture shows the instability of $\tau(\Xi_{cc}^{++})$ for large $|\psi(0)|^2$.

level (10). This contribution makes $\Gamma_{\rm SL}(\Omega_{cc}^{+})$ four times larger than $\Gamma_{\rm SL}(\Xi_{cc}^{++})$, which receives contributions only from (4) and (5). In the $\Gamma_{\rm SL}(\Xi_{cc}^{+})$, there is the Pauli interference effect at the Cabibbo suppressed level, but the rate is still made larger by 15% than that for the Ξ_{cc}^{++} baryon.

Clearly, since both semileptonic and nonleptonic rates are significantly affected by large preasymptotic effects that are proportional to $|\psi(0)|^2 \sim f_D^2$, the results for lifetimes and for the semileptonic branching ratio for Ω_{cc}^{+} depend crucially on the choice of f_D. The latter is obvious from Fig. 3, where we see that $\Gamma_{\rm SL}(\Omega_{cc}^{+})$ grows linearly with f_D^2, and especially from the second picture in Fig. 3, where it is clear that $\tau(\Xi_{cc}^{++})$ shows instability for f_D larger than 180 MeV, because of the large cancellation between the negative Pauli interference term and the contributions from (11) and (12). This is a clear signal that one should not take the results for Ξ_{cc}^{++} too literally.

Keeping the above remarks in mind, we predict the following pattern for semileptonic branching ratios:

$$BR_{\rm SL}(\Xi_{cc}^{+}) \ll BR_{\rm SL}(\Omega_{cc}^{+}) \ll BR_{\rm SL}(\Xi_{cc}^{++})\,, \qquad (18)$$

and the following pattern for the lifetimes:

$$\tau(\Xi_{cc}^{+}) \sim \tau(\Omega_{cc}^{+}) \ll \tau(\Xi_{cc}^{++})\,. \qquad (19)$$

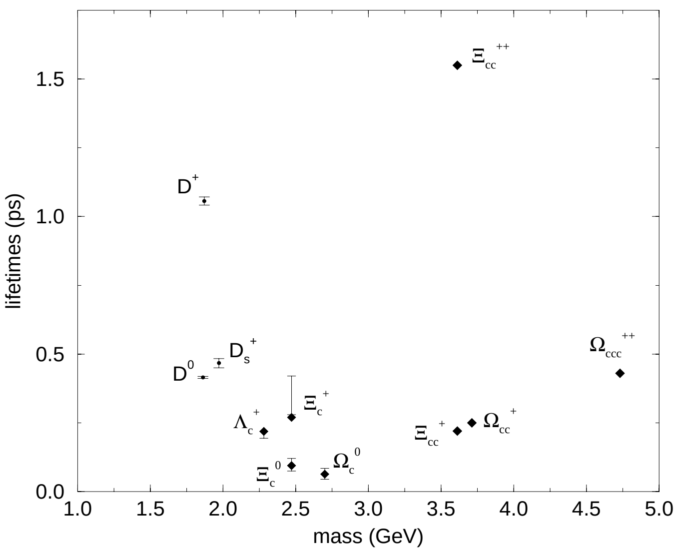

Fig. 4. Dependence of lifetimes of weakly decaying charmed hadrons on their masses. Experimental results are drawn as points with error bars (for singly-charmed hadrons), while theoretical predictions are drawn as filled diamonds (all weakly decaying charmed baryons). Values for masses of doubly- and triply-charmed baryons are taken from [29].

We can compare our results with the recent calculations of lifetimes of Ξ_{cc}^{++} and Ξ_{cc}^{+} [12]. The authors of that paper employed a similar field-theory technique, but had a different approach to the choice of relevant parameters. Throughout their paper they used the constituent heavy-quark mass as an expansion parameter, which is a phenomenological procedure that we do not find fully consistent. In the calculation of semileptonic decay rates, they did not include large preasymptotic effects, which significantly change total semileptonic widths. Comparison shows that our numerical results are significantly different from those of Kiselev, et al. [12].

Finally, it is worth discussing briefly the decay of the heaviest weakly decaying charmed hadron: the triply-charmed baryon Ω_{ccc}^{++}. Although its complicated structure and intrinsic tree-body motion prevent us from applying to this particle the heavy–light picture (as described above to other weakly decaying heavy hadrons), it is possible to give some qualitative predictions for its Ω_{ccc}^{++} decay rate and the lifetime. In this baryon, preasymptotic effects (giving large contributions in the singly-charmed and the doubly-charmed cases) do not exist, for lack of light valence quarks. Thus, the dominant contribution comes from the operators of dimensions three and five. Since in doubly-charmed decays, the contribution of dimension-five operators represents less than 20% of the contribution of the decay (dimension-three) operator, it seems reasonable to approximate the total decay width of Ω_{ccc}^{++} with the triple-c quark-decay contribution and to estimate the error of disregarding dimension-five operators at the level of 20%. In this case, the expression for $\Gamma_{\mathrm{TOT}}(\Omega_{ccc}^{++})$ can be obtained by multiplying the expressions (4), (5), (11) and (12) by a factor of 3/2, summing them and taking the limit $\mu_\pi^2 \to 0$ and $\mu_G^2 \to 0$. The numerical value for the lifetime is

$$\tau(\Omega_{ccc}^{++}) = 0.43\, ps \,. \tag{20}$$

As the calculation of dimension-five operator contributions in triply-charmed-baryon decay rates is beyond the scope of the present paper, this result can be considered as only qualitatively correct.

4 Conclusions

Application of the heavy-quark expansion to the problem of inclusive decays of doubly-charmed baryons enables us to give very interesting predictions for their lifetimes and semileptonic branching ratios. Large lifetime differences are present between Ξ_{cc}^{++}, on the one hand, and Ξ_{cc}^{+} and Ω_{cc}^{+}, on the other. Our numerical results pick out Ξ_{cc}^{++} as the longest living charmed particle (Fig. 4), although the numerical value for $\tau(\Xi_{cc}^{++})$ should be considered with certain reserve, for reasons already mentioned. Such a large numerical difference within the lifetime hierarchy makes these predictions suitable for testing by forthcoming experimental observation of doubly-charmed baryons. A theoretical prediction for semileptonic branching ratios is even clearer, and the hierarchy of BR_{SL} is unambiguously determined.

The total hierarchy of lifetimes for charmed hadrons is shown in Fig. 4. It is evident that charmed hadrons show a very complex pattern in the $\tau - M$ plane. One can note that doubly-charmed baryon lifetimes are comparable with those of their singly-charmed counterparts. This result opposes the naive expectation of roughly double widths in the doubly-charmed case, because there is decay of two, instead of one, c quark, or, in other words, correspondingly twice-smaller lifetimes of ccq baryons. However, the mesonic nature of doubly-charmed-baryon wave functions, where one uses the smaller f_D constant instead of its static F_D value, reduces four-quark operator contributions, increasing doubly-charmed-baryon lifetimes.

Although the c quark is considered not heavy enough to ensure a reasonable convergence of the heavy-quark expansion series, the numerical results in the singly-charmed sector show satisfactory qualitative and even quantitative agreement with experiment [11]. If future experiments concerning the doubly- (and triply-) charmed sector should show similar agreement with our theoretical predictions, that would have important implications for the role of four-quark operators and for the entire theory of heavy-quark expansion. Besides, the absence or the presence of agreement might have notable implications on the validity of the quark–hadron duality in the charmed sector.

Acknowledgements. This work was supported by the Ministry of Science and Technology of the Republic of Croatia under Contract No. 00980102.

Appendix: phase-space corrections

Phase-space corrections can be understood as the reduction of particle phase space, due to the propagation of the massive particle in the loops of diagrams describing inclusive decays. In our case, we consider the s quark to be massive, while the other particles (u,d quarks and leptons) are treated as massless. These corrections can be classified according to the type of operator diagram in which they appear and according to the number of massive quarks in the loop (or the number of them in a final state if we consider an inclusive process to be a sum of exclusive channels).

First, we shall enumerate corrections that appear in decay and dimension-five operator diagrams. From here on, $x = m_s^2/m_c^2$.

- One massive quark in the loop:

$$P_0(x) = (1-x^2)(1-8x+x^2) - 12x^2 \ln x\,, \qquad (21)$$

$$P_1(x) = (1-x)^4\,, \qquad (22)$$

$$P_2(x) = (1-x)^3\,. \qquad (23)$$

  $P_0(x)$ appears as a correction to the decay-type diagram, while $P_1(x)$ and $P_2(x)$ come as corrections to the chromomagnetic operator.
- Two massive quarks in the loop: Using the notation

$$v(x) = \sqrt{1-4x}\,, \qquad (24)$$

  we have

$$\tilde{P}_0(x) = v(x)(1-14x-2x^2-12x^3) + 24x^2(1-x^2)\ln\left(\frac{1+v(x)}{1-v(x)}\right)\,, \qquad (25)$$

$$\tilde{P}_1(x) = \frac{1}{2}(2\tilde{P}_0(x) - y\partial_y \tilde{P}_0(y)\,|_{y=x})\,, \qquad (26)$$

$$\tilde{P}_2(x) = v(x)(3x^2+\frac{x}{2}+1) - 3x(1-2x^2)\ln\left(\frac{1+v(x)}{1-v(x)}\right)\,. \qquad (27)$$

  As above, $\tilde{P}_0(x)$ appears in the decay diagram, while $\tilde{P}_1(x)$ and $\tilde{P}_2(x)$ are corrections to the dimension-five operator.

  Corrections in the paper due to one massive quark are systematically denoted by P, while those due to two massive quarks are denoted by $\tilde{P}$.

Next, we display the phase-space corrections to four-quark operators:

$$P_{\rm ex}(x) = (1-x)^2\,, \qquad (28)$$

$$P_{\rm int}(x) = (1-x)^2(1+x)\,, \qquad (29)$$

$$\tilde{P}_{\rm int}(x) = \sqrt{1-4x}\,. \qquad (30)$$

$P_{\rm ex}(x)$ appears as a correction to the exchange diagram, $P_{\rm int}(x)$ corrects for the massive quark in the interference contributions, and $\tilde{P}_{\rm int}(x)$ is a correction in the case of negative interference when there are two massive quarks in the loop [3,30,31].

References

1. B. Blok, M. Shifman, "Lifetimes of Charmed Hadrons Revised – Facts and Fancy", in *Proceedings of the Workshop on the Tau-Charm Factory, Marbella, Spain, 1993*, edited by J. Kirkby, R. Kirkby (Editions Frontiers, Gif-sur-Yvette, 1994), p. 247 TPI-MINN-93/55-T, UMN-TH-1227/93
2. M. Neubert, "B Decays and the Heavy-Quark Expansion", preprint CERN-TH/97-24 and hep-ph/9702375, to appear in *Heavy Flavors II*, edited by A.J. Buras and M. Lindner
3. I. Bigi, "The QCD Perspective on Lifetimes of Heavy Flavour Hadrons", preliminary version of Physics Report paper, UND-HEP-95-BIG02 and hep-ph/9508408
4. C. Caso et al., Eur. Phys. J. C**3** (1998)
5. B. Guberina, S. Nussinov, R.D. Peccei, R. Rückl, Phys. Lett. B**89**, 111 (1979)
6. N. Bilić, B. Guberina, J. Trampetić, Nucl. Phys. B**248**, 261 (1984)
7. M. Shifman, M. Voloshin, Yad. Phys. **41**, 463 (1985) [Sov. J. Nucl. Phys. **41**, 120 (1985)]
8. B. Guberina, R. Rückl, J. Trampetić, Z. Phys. C**33**, 297 (1986)
9. M. Shifman, M. Voloshin, ZhETF **91**, 1180 (1986) [JETP **64**, 698 (1986)
10. M. Neubert, Phys. Rep. **245**, 259 (1994); Int. J. Mod. Phys. A**11**, 4173 (1996)
11. B. Guberina, B. Melić, Eur. Phys. J. C**2**, 697 (1998)
12. V.V. Kiselev, A.K. Likhoded, A.I. Onischenko, preprint DESY-98-079 and hep-ph/9807354
13. C.S. Kim, A.D. Martin, Phys. Lett B**225**, 186 (1989)
14. Y. Nir, Phys. Lett. B**221**, 184 (1989)
15. M.B. Voloshin, Phys. Lett. B**385**, 369 (1996)
16. E. Bagan, P. Ball, V.M. Braun, P. Gosdzinsky, Nucl. Phys. B**432**, 3 (1994)
17. M. Beneke, G. Buchalla, Phys. Rev. D**53**, 4991 (1996)
18. G.P. Lepage, L. Magnea, C. Nakhleh, U. Magnea, K. Hornbostel, Phys. Rev. D**46**, 4052 (1992)
19. W.E. Caswell, G.P. Lepage, Phys. Lett. B**167**, 437 (1986)
20. M.J. Savage, M.B. Wise, Phys. Lett. B**248**, 177 (1990)
21. M.J. White, M.J. Savage, Phys. Lett. B**271**, 410 (1991)
22. M.J. Savage, R.P. Springer, Int. J. Mod. Phys. A**6**, 1701 (1991)
23. A. De Rújula, H. Georgi, S.L. Glashow, Phys. Rev. D**12**, 147 (1975)
24. J.L. Cortes, J. Sanchez Guillen, Phys. Rev. D**24**, 2982 (1981)
25. H. Wittig, Int. J. Mod. Phys. A**12**, 4477 (1997)
26. M. Neubert, Phys. Rev. D**45**, 2451 (1992)
27. S. Narrison, Phys. Lett. B**322**, 247 (1994)
28. D. Ebert, R.N. Faustov, V.O. Galkin, A.P. Martynenko, V.A. Saleev, Z. Phys. C**76**, 111 (1997)
29. J.G. Körner, D. Pirjol, M. Krämer, Prog. Part. Nucl. Phys. **33**, 787 (1994)
30. J.L. Cortes, X.Y. Pham, A. Tounsi, Phys. Rev. D**25**, 188 (1982)
31. Q. Hokim, X.Y. Pham, Phys. Lett. B**122**, 297 (1983)